JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 19, No. 4, July–August 1996

Adaptive Control of Feedback Linearizable Nonlinear Systems with Application to Flight Control

Sahjendra N. Singh*
University of Nevada, Las Vegas, Nevada 89154-4026
and
Marc Steinberg†
U.S. Naval Air Warfare Center, Warminster, Pennsylvania 18974

The question of output trajectory control of a class of input–output feedback linearizable nonlinear dynamical systems using state variable feedback in the presence of parameter uncertainty is considered. For the derivation of a control law, a hypersurface is chosen, which is a linear function of the tracking error, its derivatives, and the integral of the tracking error. An adaptive control law is derived such that in the closed-loop system the trajectory asymptotically converges to this hypersurface. For any trajectory evolving on this surface, the tracking error tends to zero. Based on these results, a new approach to the design of an adaptive flight control system is presented. In the closed-loop system, trajectory control of the sets of output variables roll angle, angle of attack, and sideslip angle (ϕ, α, β) and roll, pitch, and sideslip angles (ϕ, θ, β) using aileron, rudder, and elevator control is presented. Simulation results are obtained to show that precise simultaneous longitudinal and lateral maneuvers can be performed in spite of large uncertainty in the aerodynamic parameters.

I. Introduction

GEOMETRIC nonlinear control theory has provided powerful tools for systematic design of nonlinear feedback systems.[1,2] For nonlinear tracking control system design, mostly input–output and exact linearization techniques are used. These approaches are limited to systems with known parameters. Recent advances in geometric nonlinear adaptive control theory have provided techniques for designing controllers for nonlinear uncertain systems.[3–7] The procedures for adaptive controller design in Refs. 3–5, however, introduce new parameters requiring a large-order controller. In a recent interesting paper, Kristic et al.[6] do not introduce new parameters for the derivation of controller and following their approach, one obtains an adaptive controller of the same order as the number of actual unknown parameters in the model. A neural controller[8] has been also designed for a class of nonlinear systems with known input matrix using the results of Ref. 6.

Modern high-performance aircraft operate in a large flight envelope. Maneuvers such as rolling pullouts and high-acceleration turns require a simultaneous rapid rolling and pitching motion. The dynamical model of aircraft has significant nonlinearity during such maneuvers. In recent years, inverse and nonlinear flight controllers have been designed for performing large maneuvers.[9–16] These results are based on the assumption that the aerodynamic parameters are known. Attempt has been also made to analyze the robustness of inverse controllers and design robust inverse flight controllers.[17–19] However, enhancement of robustness in these papers is based on μ synthesis, which requires linearization of the model and, therefore, is useful for only small uncertainty in the system parameters. It appears from the literature that the design of adaptive controllers for nonlinear models of aircraft performing roll-coupled maneuvers in the presence of large uncertainty has not gained attention.

The contribution of this paper lies in derivation of an adaptive controller for a class of feedback linearizable nonlinear systems and an application to flight control in the presence of large parameter uncertainty. The controller derivation procedure is based on a backstepping design technique used in Ref. 6. However, we obtain a new control law based on a different choice of Lyapunov function. In Ref. 6, a quadratic Lyapunov function is used for design. Here, instead, a Lyapunov function is chosen to regulate the trajectory to a chosen hypersurface $s = 0$ in the state space. The function s is a linear combination of the tracking error, its higher-order derivatives, and its integral. Such a choice of hypersurface is motivated by its use in sliding mode and flight control designs.[20,21] This choice of hypersurface allows filtering of the tracking error by a stable filter and enhances the robustness of the control system because of integral feedback. For any trajectory confined to this surface, the tracking error tends to zero. Based on these results, a new approach to the design of a flight controller for nonlinear roll-coupled maneuvers of aircraft using aileron, rudder, and elevator control is presented. Simulation results for the trajectory control of the sets of output variables roll angle, angle of attack, and sideslip angle (ϕ, α, β) and roll, pitch, and sideslip angles (ϕ, θ, β) are obtained to show the capability of the controller.

This paper is organized as follows. Section II presents the control problem. An adaptive control law is derived in Sec. III. The control of aircraft is considered in Secs. IV and V, and Sec. VI presents the numerical results.

II. Problem Formulation

We shall consider a class of systems described by

$$\begin{aligned} \dot{x} &= f(x, w) + g(x, w_u)u \\ y &= c(x) \end{aligned} \tag{1}$$

where the state vector $x \in M$, a compact subset of R^n; the control input $u \in U \subset R^m$; the output $y = [y_1, \ldots, y_m]^T \in R^m$; $(w, w_u) \in W \times W_u \subset R^{k_1} \times R^{k_2}$ is the vector of unknown parameters; $w = (w_1^T, w_2^T)^T \in W \subset R^{k_{11}} \times R^{k_{12}}$; and T denotes transposition. Here the number of outputs is same as the number of inputs, since m inputs can control only m outputs independently. The functions $f(x, w)$ and $c(x) = [c_1(x), \ldots, c_m(x)]^T$ are assumed to be continuously differentiable sufficient number of times on M, and $g(x, w_u) = [g_1(x, w_u), \ldots, g_m(x, w_u)]$ is a continuously differentiable function of x. (For compactness in notation often the arguments of various functions are suppressed.) The functions f and g are assumed to depend linearly on the unknown parameters.

We assume that the class of systems (1) is input–output feedback linearizable.[1,2] Define the Lie derivatives of $c_i(x)$ and any smooth function $\kappa(x)$ with respect to $f(x, w)$, and $g(x, w_u)$

Received July 15, 1995; revision received Dec. 13, 1995; accepted for publication Dec. 17, 1995.

*Professor, Department of Electrical and Computer Engineering. Associate Fellow AIAA.

†Flight Control Engineer, Aircraft Division, Code 6053. Member AIAA.

as $L_f\kappa = (\partial\kappa/\partial x)f$, $L_g\kappa = (L_{g_1}\kappa, \ldots, L_{g_m}\kappa)$, and $L_g c = [(L_g c_1)^T, \ldots, (L_g c_m)^T]^T$. Let the relative degree of y_i be $(r_i + 1)$. The relative degree $(r_i + 1)$ is defined to be the least nonnegative integer j such that the jth derivative $y_i^{(j)} = \mathrm{d}^j y_i/\mathrm{d}t^j$ of y_i along the trajectory of system (1) explicitly depends on u for the first time.[1] Then, derivatives of y_i are given by $(i = 1, \ldots, m)$

$$
\begin{aligned}
y_i^{(j)} &= L_f^j c_i, \qquad j = 1, \ldots, r_i \\
y_i^{(r_i+1)} &= L_f^{r_i+1} c_i + \left(L_g L_f^{r_i} c_i\right) u
\end{aligned}
\tag{2}
$$

If the decoupling matrix $E = [(L_g L_f^{r_1} c_1)^T, \ldots, (L_g L_f^{r_m} c_m)^T]^T$ is nonsingular on M, then the input–output map of system (1) is invertible, and one can design a decoupling controller for the control of output vector provided the system parameters are known.

For the system (1) with uncertain parameters we assume that the following conditions are satisfied.

Assumption 1: There exist vectors $x_1 \in R^{n-m}$ and $\omega \in R^m$ formed by a selection of the components of the state vector x; functions $\phi_{i0}(x_1)$; vector $\psi_0(x)$; row vectors $\phi_i(x_1)$ and $B_i(x_1)$; and matrices $\psi_1(x)$ and $D(x, w_u)$ such that $(i = 1, \ldots, m)$.

1) $L_f^k c_i$, $j = 0, 1, \ldots, r_i - 1$ are only functions of x_1 and are independent of parameter vector w.

2) $L_g\phi_{i0} = 0$, $L_g(\phi_i)^T = 0$, and $L_g(B_i)^T = 0$.

3) $L_f^{r_i} c_i = \phi_{i0}(x_1) + \phi_i(x_1)w_1 + B_i(x_1)\omega$.

4) $\dot\omega = \psi_0(x) + \psi_1(x)w_2 + D(x, w_u)u$.

We have chosen ω to denote a subvector of x since this is used later to represent the vector of angular velocities of aircraft. Define $\Phi_0(x_1) = (\phi_{10}, \ldots, \phi_{m0})^T$. The chosen m vector ω will be used as an input to control the m output vector y, and m inputs u will control m vector ω independently. Define $\Phi_0(x_1) = (\phi_{10}, \ldots, \phi_{m0})^T$, $\Phi(x_1) = (\phi_1^T, \ldots, \phi_m^T)^T$, and $B(x_1) = [B_1^T(x_1), \ldots, B_m^T(x_1)]^T$.

Assumption 2: The $m \times m$ matrices $D(x, w_u)$ and $B(x_1)$ are invertible on $M \times W_u$.

According to these assumptions, it easily follows that the decoupling matrix is $E = BD$, and system (1) is input–output feedback linearizable. For input–output linearizable systems, one can define a local diffeomorphism $P\colon M \to M^1$:

$$
\begin{aligned}
P(x) = \big[&L_f^0 c_1, \ldots, L_f^{r_1-1} c_1; \ldots; L_f^0 c_m, \ldots, L_f^{r_m-1} c_m; \\
&\omega^T; \gamma^T(x)\big]^T \in R^n
\end{aligned}
\tag{3}
$$

where the new coordinate vector $\gamma \in R^{n_0}$, $n_0 = n - \sum_{i=1}^m r_i - m$. Define a new state vector $\xi = (z^T, \omega^T, \eta^T)^T = P(x)$, where $z = (z_1^T, \ldots, z_m^T)^T$, $z_i = (z_{i1}, \ldots, z_{ir_i})^T = (L_f^0 c_i, \ldots, L_f^{r_i-1} c_i)^T$, and $\eta = \gamma$, to obtain a new representation of system (1) given by $(i = 1, \ldots, m)$

$$
\begin{aligned}
\dot z_{ik} &= z_{i,(k+1)}, \qquad k = 1, \ldots, r_i - 1 \\
\dot z_{ir_i} &= \phi_{i0}(x_1) + \phi_i(x_1)w_1 + B_i(x_1)\omega \\
\dot\omega &= \psi_0(\xi) + \psi_1(\xi)w_2 + D(\xi, w_u)u \\
\dot\eta &= q_0(\xi) + q_1(\xi)w + q_2(\xi, w_u)u
\end{aligned}
\tag{4}
$$

where $x = P^{-1}(\xi)$, and $q_i(\xi)(i = 0, 1, 2)$ are appropriate matrices.

For trajectory tracking, smooth reference trajectories $y_c(t)$ are generated by command generators of the form $(i = 1, \ldots, m)$

$$
\Pi_i(\hat D)\left[y_{ci}(t) - y_i^*\right] = 0 \tag{5}
$$

where $\hat D = \mathrm{d}/\mathrm{d}t$; $\Pi_i(\lambda) = \lambda^{r_i+2} + p_{i(r_i+1)}\lambda^{r_i+1} + \cdots + p_{i1}\lambda + p_{i0}$ is a Hurwitz polynomial for a proper choice of parameters p_{ik} and y^* is the desired terminal value of y. The order the system (5) is chosen to be is $r_i + 2$ since $y_{ci}^{(r_i+1)}$ will be needed in the control law.

We are interested in deriving an adaptive control law $u(x, t)$ such that in the closed-loop system $y(t)$ tracks $y_c = [y_{c1}, \ldots, y_{cm}]^T$ in spite of the uncertainty in parameter vector (w, w_u), that is, the tracking error $e(t) = [e_1, \ldots, e_m]^T = y(t) - y_c(t)$ tends to zero as $t \to \infty$.

III. Adaptive Control

In this section an adaptive control law for the tracking of y_c is derived. Define a vector function $s = (s_1, \ldots, s_m)^T$, where for $i = 1, \ldots, m$,

$$
\begin{aligned}
s_i &= e_i^{(r_i-1)} + k_{ir_i-1}e_i^{(r_i-2)} + \cdots + k_{i1}e_i + k_{i0}x_{si} \\
\dot x_{si} &= e_i
\end{aligned}
\tag{6}
$$

where the gains k_{ij} are positive constants chosen such that the polynomial

$$
\mu_i(\lambda) = \lambda^{r_i} + k_{i(r_i-1)}\lambda^{r_i-1} + \cdots + k_{i1}\lambda + k_{i0} = \prod_{j=1}^{r_i}(\lambda - \lambda_{ij}) \tag{7}
$$

is Hurwitz. The function s_i is a linear combination of error e_i, its derivatives, and includes an integral term as well. Using Eq. (2), Eq. (6) can be written as

$$
s_i = \sum_{j=1}^{r_i} k_{ij}\left[L_f^{j-1}(c_i) - y_{ci}^{(j-1)}\right] + k_{i0}x_{si} \tag{8}
$$

where $k_{ir_i} = 1$. In view of Eq. (8), one observes that if $s_i(t) \equiv 0$, then $e_i(t)$ asymptotically tends to zero.

Differentiating Eq. (8) and using Eq. (4) gives

$$
\begin{aligned}
\dot s_i &= \phi_i(x_1)w_1 + B_i(x_1)\omega \\
&\quad + \left[\phi_{i0}(x_1) - y_{ci}^{(r_i)} + \sum_{j=0}^{r_i-1} k_{ij}\left(L_f^j(c_i) - y_{ci}^{(j)}\right)\right] \\
&\doteq \phi_i(x_1)w_1 + B_i(x_1)\omega + v_i(x_1, x_c)
\end{aligned}
\tag{9}
$$

where v_i is the term in the square bracket in Eq. (9) and $x_c = [y_{c1}, \ldots, y_{c1}^{(r_1+2)}; \ldots; y_{cm}, \ldots, y_{cm}^{(r_m+2)}) \in R^{m(r_i+2)}$ is the state vector associated with the command generator (5).

Define $\tilde\omega = \omega - \omega_d$, the parameter error vector $\tilde w = w - \hat w = [(w_1 - \hat w_1)^T, (w_2 - \hat w_2)^T]^T$ and $\tilde w_u = w_u - \hat w_u$, where ω_d is to be determined later and $\hat w_i$ and $\hat w_u$ are the estimates of w_i and w_u, respectively. Let $v = (v_1, \ldots, v_m)^T$. Using Eq. (9), one obtains

$$
\dot s = \Phi(x_1)w_1 + B(x_1)(\tilde\omega + \omega_d) + v(x_1, x_c) \tag{10}
$$

For deriving the adaptive controller, the Lyapunov approach, together with the recursive backstepping design procedure introduced in Refs. 5 and 6, is used. First we consider the stabilization of subsystem (10), treating ω_d as an input. For this, consider a quadratic Lyapunov function

$$
U_1 = \left(s^T s + \tilde w_1^T L_1 \tilde w_1\right)/2 \tag{11}
$$

The matrix L_1 is a positive definite symmetric matrix (denoted as $L_1 > 0$). Differentiating U_1 along the trajectory of system (4), one has

$$
\dot U_1 = s^T[\Phi(x_1)w_1 + B(x_1)(\tilde\omega + \omega_d) + v(x_1, x_c)] + \tilde w_1^T L_1 \dot{\tilde w}_1 \tag{12}
$$

In view of Eq. (12), choosing ω_d to cancel the known and estimated terms gives

$$
\omega_d = B^{-1}(x_1)[-c_{11}s - \Phi(x_1)\hat w_1 - v(x_1, x_c)] \tag{13}
$$

where c_{11} is a positive constant. Substituting Eq. (13) in Eq. (12) gives

$$
\dot U_1 = -c_{11}s^T s + \tilde w_1^T\left[\Phi^T(x_1)s + L_1\dot{\tilde w}_1\right] + \tilde\omega^T B^T(x_1)s \tag{14}
$$

Substituting Eq. (13) in Eq. (10) gives

$$
\dot s = -c_{11}s + \Phi(x_1)\tilde w_1 + B(x_1)\tilde\omega \tag{15}
$$

Differentiating Eq. (13) gives

$$
\dot\omega_d = L_f\omega_d + L_g\omega_d u + \left(\frac{\partial\omega_d}{\partial x_c}\right)\dot x_c + \left(\frac{\partial\omega_d}{\partial s}\right)\dot s + \left(\frac{\partial\omega_d}{\partial\hat w_1}\right)\dot{\hat w}_1 \tag{16}
$$

In view of Assumption 1 (part 2) and the function $\boldsymbol{v}$ defined in Eq. (9), one has $L_g\omega_d = 0$. Noting that the highest-order derivative of y_{ci} in v_i is r_i, and the parameters appear linearly in $L_f\omega_d$, there exist matrices ψ_{0d}, ψ_{1d}, and ψ_{2d} such that

$$\dot{\omega}_d = \psi_{0d}(\boldsymbol{x}, \boldsymbol{x}_s, \boldsymbol{x}_c, \hat{\boldsymbol{w}}_1, \dot{\hat{\boldsymbol{w}}}_1) - \psi_{1d}(\boldsymbol{x}, \boldsymbol{x}_c, \hat{\boldsymbol{w}}_1)\boldsymbol{w}_1 + \psi_{2d}(\boldsymbol{x}, \boldsymbol{x}_c, \hat{\boldsymbol{w}}_1)\boldsymbol{w}_2 \tag{17}$$

Using Eqs. (4) and (17), the derivative of $\tilde{\omega}$ can be written as

$$\begin{aligned}\dot{\tilde{\omega}} &= (\psi_0 - \psi_{0d}) + \psi_{1d}\boldsymbol{w}_1 + (\psi_1 - \psi_{2d})\boldsymbol{w}_2 + D(\boldsymbol{x}, \boldsymbol{w}_u)\boldsymbol{u} \\ &\doteq \psi_{0a}(\boldsymbol{x}, \boldsymbol{x}_s, \boldsymbol{x}_c, \hat{\boldsymbol{w}}, \dot{\hat{\boldsymbol{w}}}_1) + \psi_{1a}\tilde{\boldsymbol{w}}_1 + \psi_{2a}\tilde{\boldsymbol{w}}_2 + D(\boldsymbol{x}, \boldsymbol{w}_u)\boldsymbol{u}\end{aligned} \tag{18}$$

where $\psi_{0a} = \psi_0 - \psi_{0d} + \psi_{1d}\hat{\boldsymbol{w}}_1 + \psi_{2a}\hat{\boldsymbol{w}}_2$, $\psi_{1a} = \psi_{1d}$, and $\psi_{2a} = \psi_1 - \psi_{2d}$.

Now for the stabilization of the complete system (15) and (18), consider the modified Lyapunov function

$$U_2 = U_1 + \left(\tilde{\omega}^T\tilde{\omega} + \tilde{\boldsymbol{w}}_2^T L_2 \tilde{\boldsymbol{w}}_2 + \tilde{\boldsymbol{w}}_u^T L_3 \tilde{\boldsymbol{w}}_u\right)/2 \tag{19}$$

where matrices $L_2 > 0$ and $L_3 > 0$. Differentiating Eq. (19) and using Eqs. (14) and (18) gives

$$\dot{U}_2 = -c_{11}\boldsymbol{s}^T\boldsymbol{s} + \tilde{\boldsymbol{w}}_1^T\left[\Phi^T\boldsymbol{s} + L_1\dot{\tilde{\boldsymbol{w}}}_1 + \psi_{1a}^T\tilde{\omega}\right] + \tilde{\boldsymbol{w}}_2^T\left[L_2\dot{\tilde{\boldsymbol{w}}}_2 + \psi_{2a}^T\tilde{\omega}\right] + \tilde{\boldsymbol{w}}_u^T L_3\dot{\tilde{\boldsymbol{w}}}_u + \tilde{\omega}^T\left[\boldsymbol{B}^T\boldsymbol{s} + \psi_{0a} + D\boldsymbol{u}\right] \tag{20}$$

In view of Eq. (20), choosing the control and parameter update laws to cancel known and estimated functions, one obtains

$$\begin{aligned}\boldsymbol{u} &= \hat{D}^{-1}\left(-\boldsymbol{B}^T\boldsymbol{s} - \psi_{0a} - c_{22}\tilde{\omega}\right) \\ \dot{\tilde{\boldsymbol{w}}}_1 &= -\dot{\hat{\boldsymbol{w}}}_1 = -L_1^{-1}\left(\Phi^T\boldsymbol{s} + \psi_{1a}^T\tilde{\omega}\right) \\ \dot{\tilde{\boldsymbol{w}}}_2 &= -\dot{\hat{\boldsymbol{w}}}_2 = -L_2^{-1}\psi_{2a}^T\tilde{\omega}\end{aligned} \tag{21}$$

where c_{22} is a positive number and $\hat{D} = D(\boldsymbol{x}, \hat{\boldsymbol{w}}_u)$. Substituting Eq. (21) in Eq. (20) gives

$$\dot{U}_2 = -c_{11}\boldsymbol{s}^T\boldsymbol{s} - c_{22}\tilde{\omega}^T\tilde{\omega} + \tilde{\boldsymbol{w}}_u^T L_3\dot{\tilde{\boldsymbol{w}}}_u + \tilde{\omega}^T(D - \hat{D})\boldsymbol{u} \tag{22}$$

Since parameters appear linearly in Eq. (1), there exists a matrix Ψ_u such that

$$[D(\boldsymbol{x}, \boldsymbol{w}_u) - D(\boldsymbol{x}, \hat{\boldsymbol{w}}_u)]\boldsymbol{u} = \Psi_u(\boldsymbol{x}, \boldsymbol{u})\tilde{\boldsymbol{w}}_u \tag{23}$$

Substituting Eq. (23) in Eq. (22), gives

$$\dot{U}_2 = -c_{11}\boldsymbol{s}^T\boldsymbol{s} - c_{22}\tilde{\omega}^T\tilde{\omega} + \tilde{\boldsymbol{w}}_u^T\left[L_3\dot{\tilde{\boldsymbol{w}}}_u + \Psi_u^T(\boldsymbol{x}, \boldsymbol{u})\tilde{\omega}\right] \tag{24}$$

Finally, the update law for $\hat{\boldsymbol{w}}_u$ is selected as

$$\dot{\tilde{\boldsymbol{w}}}_u = -\dot{\hat{\boldsymbol{w}}}_u = -L_3^{-1}\Psi_u^T(\boldsymbol{x}, \boldsymbol{u})\tilde{\omega} \tag{25}$$

Substituting Eq. (25) in Eq. (24) gives

$$\dot{U}_2 = -c_{11}\boldsymbol{s}^T\boldsymbol{s} - c_{22}\tilde{\omega}^T\tilde{\omega} \le 0 \tag{26}$$

Since $\dot{U}_2$ is negative semidefinite, it follows that $\boldsymbol{s}$, $\tilde{\omega}$, $\tilde{\boldsymbol{w}}_i$, and $\tilde{\boldsymbol{w}}_u$ are bounded. Noting that $\mu_i(\lambda)$ is a Hurwitz polynomial, boundedness of s_i implies boundedness of $e_i^{(k)}$ in Eq. (6) and z_i since $\boldsymbol{x}_c$ is bounded. We assume in the following that for the chosen command trajectory $\boldsymbol{y}_c$, ω_d, and $\eta(t)$ are bounded. (Such an assumption on the remaining subvector η of $\boldsymbol{\xi}$ is common in literature in output tracking problems.[3,5,7]) Thus $\boldsymbol{\xi}$ and $\boldsymbol{x}$ are bounded, and we assume that $\boldsymbol{x}$ remains in M. Then by smoothness of functions Φ, ψ_{ik}, $\boldsymbol{B}$, and D, using Eqs. (15) and (18), it is seen that $\dot{\tilde{\omega}}$ and $\dot{\boldsymbol{s}}$ are bounded. Now taking the derivative of $\dot{U}_2$, one observes that $\ddot{U}_2$ is bounded. Since $\dot{U}_2 \le 0$ and U_2 is lower bounded, it follows that U_2 has a limit as $t \to \infty$. Now for obtaining stability result, one invokes Barbalat's lemma,[20] which states that if a differentiable function $g(t)$ has a finite limit as $t \to \infty$ and is such that $\ddot{g}(t)$ exists and is bounded, then $\dot{g}(t) \to 0$ as $t \to \infty$. Since U_2 satisfies the hypotheses of lemma, it follows that $\dot{U}_2 \to 0$. Thus in view of Eq. (26), this implies that $\boldsymbol{s}(t)$ and $\tilde{\omega}$ tend to zero as $t \to \infty$. In view of Eq. (6), the transfer function relating e_i as an output and s_i as an input is given by

$$F_i(\tilde{s}) = \frac{\tilde{s}}{\tilde{s}^{r_i} + k_{ir_i-1}\tilde{s}^{r_i-1} + \cdots + k_{i1}\tilde{s} + k_{i0}} \tag{27}$$

where $\tilde{s}$ is the Laplace variable. Since $F_i(\tilde{s})$ is a strictly stable filter and the input $s_i(t)$ to the filter converges to zero, it follows that e_i also tends to zero.

The feedback gains k_{ij} provide additional flexibility in shaping the tracking error responses. In view of Eq. (27), it follows that if $\boldsymbol{s}(t)$ tends to a constant function under the influence of certain unmodeled disturbance in Eq. (1), then $\boldsymbol{e}(\infty) = 0$ due to error integral feedback. Thus integral feedback enhances the robustness of controller.

Remark 1: It is assumed here that $\hat{\boldsymbol{w}}_u$ remains in W_u. Thus, $\hat{D}$ is invertible, and the control law (21) is well defined. In case of parameter drift due to unmodeled dynamics of the system, one may use a modified version of update law based on projection algorithm to confine $\hat{\boldsymbol{w}}_u$ to the set W_u, or introduce σ modification in the update law.[22]

Remark 2: Zero dynamics describe the internal dynamics of the closed-loop system when the tracking error $\boldsymbol{e}$ is identically zero (Ref. 1, pp. 175–182). The state component η is associated with the zero dynamics of the system. For satisfactory output tracking performance, η must remain bounded in the closed-loop system. This is accomplished by a proper selection of output variables for controller design such that the zero dynamics are stable or have bounded trajectories.

IV. Aircraft Model

For studying the dynamics of the aircraft, its principal axes are chosen as body axes. The complete set of equations of motion of the aircraft is given by (see Refs. 23 and 24 for the derivation and the notation)

$$\begin{pmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\\ \dot{\alpha}\\ \dot{\beta}\\ \dot{\phi}\\ \dot{\theta}\end{pmatrix} = \begin{pmatrix} l_\beta\beta + l_q q + l_r r + (l_{\beta\alpha}\beta + l_{r\alpha}r)\Delta\alpha + l_p p - i_1 qr \\ \bar{m}_\alpha\Delta\alpha + \bar{m}_q q + i_2 pr - m_{\dot{\alpha}}p\beta + m_{\dot{\alpha}}(g_0/V)(\cos\theta\cos\phi - \cos\theta_0) \\ n_\beta\beta + n_r r + n_p p + n_{p\alpha}p\Delta\alpha - i_3 pq + n_q q \\ q - p\beta + z_\alpha\Delta\alpha + (g_0/V)(\cos\theta\cos\phi - \cos\theta_0) \\ y_\beta\beta + p(\sin\alpha_0 + \Delta\alpha) - r\cos\alpha_0 + (g_0/V)\cos\theta\sin\phi \\ p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \\ q\cos\phi - r\sin\phi \end{pmatrix} + \begin{pmatrix} \tilde{l}_{\delta a} & l_{\delta r} & 0 \\ 0 & 0 & \bar{m}_{\delta e} \\ \tilde{n}_{\delta a} & n_{\delta r} & 0 \\ 0 & 0 & z_{\delta e} \\ y_{\delta a} & y_{\delta r} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix}\delta a\\ \delta r\\ \delta e\end{pmatrix} \triangleq \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{w}) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{w}_u)\boldsymbol{u} \tag{28}$$

where the aerodynamic parameter vectors $\boldsymbol{w}$ and $\boldsymbol{w}_u$ are defined later, $\Delta\alpha = \alpha - \alpha_0$, the state vector $\boldsymbol{x} = [p, q, r, \alpha, \beta, \phi, \theta]^T$, the control vector $\boldsymbol{u} = [\delta a, \delta r, \delta e]^T$, $\tilde{l}_{\delta a} = l_{\delta a} + l_{\alpha\delta a}\Delta\alpha$, and $\tilde{n}_{\delta a} = n_{\delta a} + n_{\alpha\delta a}\Delta\alpha$. Here the aerodynamic functions are assumed be linearly dependent on parameters $\boldsymbol{w}$ and $\boldsymbol{w}_u$. This is possible since any nonlinear function can be always expressed as a linear combination of suitably chosen basis functions of $\boldsymbol{x}$.

Here the speed V is assumed constant, and the model contains only a rudimentary representation of the aerodynamic nonlinearities. The assumption of a constant speed in large maneuver may seem unrealistic. However, this simplification is not essential since throttle control can be considered as an additional input for controlling the speed if V is treated as an output variable. The design approach presented here is applicable to models with complete aerodynamic nonlinearities and actuator dynamics. Inclusion of these nonlinearities and the actuator dynamics simply increases the computational difficulty.

V. Adaptive Flight Control System

In this section a flight control system is designed. Suppose that a reference trajectory $y_c = (\phi_c, \alpha_c, \beta_c)^T$ is generated by the command generator (5) with $y^* = (\phi^*, \alpha^*, \beta^*)^T$, a given terminal value of the output where $y = c(x) = (\phi, \alpha, \beta)^T$. It is desired to design a flight controller so that y tracks y_c in spite of large uncertainty in the parameters w and w_u.

Since aileron, rudder, and elevator are principally moment producing devices, the elements $y_{\delta a}$, $y_{\delta r}$, and $z_{\delta e}$ are quite small. For the controller design, we neglect these terms by setting $y_{\delta a} = y_{\delta r} = z_{\delta e} = 0$ in $\dot{y}$. Thus the derivatives of y are given by

$$\dot{y} = L_f c$$
$$\ddot{y} = L_f^2 c + (L_g L_f c)u \tag{29}$$

Since $L_g L_f c_i \neq 0$, $(i = 1, 2, 3)$, $r_i + 1 = 2$, and the aircraft model is input–output feedback linearizable.

We shall verify that the aircraft model can be represented by Eq. (4). Let $\xi = (z^T, \omega^T, \eta)^T$, where $z = y = (\phi, \alpha, \beta)^T$, the angular velocity vector $\omega = (p, q, r)^T$, $\eta = \theta$, and $x_1 = (\alpha, \beta, \phi, \theta)^T$. Then the differential equation for z given by

$$\begin{pmatrix}\dot{\phi}\\ \dot{\alpha}\\ \dot{\beta}\end{pmatrix} = \begin{bmatrix}0\\ z_\alpha \Delta\alpha + (g_0/V)(\cos\theta\cos\phi - \cos\theta_0)\\ y_\beta\beta + (g_0/V)\cos\theta\sin\phi\end{bmatrix} + \begin{bmatrix}1 & \tan\theta\sin\phi & \tan\theta\cos\phi\\ -\beta & 1 & 0\\ (\sin\alpha_0 + \Delta\alpha) & 0 & -\cos\alpha_o\end{bmatrix}\begin{pmatrix}p\\ q\\ r\end{pmatrix}$$
$$\triangleq \Phi_0(x_1) + \Phi(x_1)w_1 + B(x_1)\omega \tag{30}$$

has a form similar to that described in Eq. (4), where $w_1 = (w_{y1}, w_{y2})^T = (z_\alpha, y_\beta)^T \in W_1 \subset R^2$. The matrices Φ and B are easily determined by comparing terms in Eq. (30), and

$$\Phi_0 = \left[0, g_0 V^{-1}(\cos\theta\cos\phi - \cos\theta_0), g_0 V^{-1}\cos\theta\sin\phi\right]^T$$

is the vector independent of state ω and parameter vector w. The matrix B is nonsingular at each $x \in M_1$, where

$$M_1 = \{x \in R^7\colon \cos\alpha_0 + \tan\theta[(\sin\alpha_0 + \Delta\alpha)\cos\phi + \beta\sin\phi\cos\alpha_0] \neq 0\} \tag{31}$$

Define the parameter vectors as $w_2 = (w_p^T, w_q^T, w_r^T)^T \in W_2 \subset R^{14}$, $w_u = (w_{u1}^T, w_{u2}^T, w_{u3}^T)^T \in W_u \subset R^7$, $w_{u1} = (l_{\delta a}, l_{\alpha\delta a}, l_{\delta r})^T$, $w_{u2} = \bar{m}_{\delta e}$, and $w_{u3} = (n_{\delta a}, n_{\alpha\delta a}, n_{\delta r})^T$, where

$$w_p = (l_\beta, l_p, l_q, l_r, l_{\beta\alpha}, l_{r\alpha})^T \in R^6$$
$$w_q = (\bar{m}_\alpha, \bar{m}_q, m_{\dot{\alpha}})^T \in R^3 \tag{32}$$
$$w_r = (n_\beta, n_r, n_p, n_{p\alpha}, n_q)^T \in R^5$$

Here $W_i (i = 1, 2)$ and W_u are some compact subsets to which the aerodynamic parameters belong. The parameter vector $w_2 = (w_p^T, w_q^T, w_r^T)^T$ and $w_u = (w_{u1}^T, w_{u2}, w_{u3}^T)^T$ are the aerodynamic parameters in the differential equation for p, q, and r, and the parameter vector w_1 is present in the differential equation for α and β. We define functions $\psi_0 = (-i_1 qr, i_2 pr, -i_3 pq)^T$, $\psi_1 = \mathrm{diag}(\psi_p, \psi_q, \psi_r) \in R^{3\times 14}$, where

$$\psi_p = (\beta, p, q, r, \beta\Delta\alpha, r\Delta\alpha)$$
$$\psi_q = [\Delta\alpha, q, (g_0/V)(\cos\theta\cos\phi - cos\theta_0) - p\beta] \tag{33}$$
$$\psi_r = (\beta, r, p, p\Delta\alpha, q)$$

$\Phi(x_1)$ and $\psi_1(x)$ are the regression matrices, which appear in the differential equation for y and $\omega = (p, q, r)^T$, respectively, as factors of unknown parameters, and Φ_0 and ψ_0 are known functions. Using these definitions, the differential equation for ω is

$$\dot{\omega} = \psi_0(x) + \mathrm{diag}(\psi_p, \psi_q, \psi_r)w_2 + D(x, w_u)u \tag{34}$$

where

$$D(x, w_u) = \begin{pmatrix}\tilde{l}_{\delta a} & l_{\delta r} & 0\\ 0 & 0 & \bar{m}_{\delta e}\\ \tilde{n}_{\delta a} & n_{\delta r} & 0\end{pmatrix}$$

We note that Eq. (34) has a form similar to that given in Eq. (4). The rank of D is 3 at each $(x, w_u) \in M \times W_u$, where

$$M = \{x \in M_1 : (\tilde{l}_{\delta a} n_{\delta r} - \tilde{n}_{\delta a} l_{\delta r})\bar{m}_{\delta e} \neq 0, w_u \in W_u\} \tag{35}$$

We shall be interested in trajectories evolving on M.

Since $r_i = 1$, the function s chosen according to Eq. (6) is given by

$$s = e + K_0 x_s \qquad \dot{x}_s = e \tag{36}$$

where the tracking error $e = (e_1, e_2, e_3)^T = [(\phi - \phi_c), (\alpha - \alpha_c), (\beta - \beta_c)]^T$ and $K_0 = \mathrm{diag}(k_{10}, k_{20}, k_{30})$. Differentiating s and using Eq. (9) gives

$$\dot{s} = (L_f c - \dot{y}_c) + K_0 e$$
$$\doteq \Phi w_1 + B\omega + v \tag{37}$$

where $v = \Phi_0 - \dot{y}_c + K_0 e$. Now one computes ω_d using Eq. (13) and $\psi_{ka} (k = 0, 1, 2)$ using Eq. (18). The 3×7 matrix $\Psi_u(x, u) = \mathrm{diag}(\psi_{up}, \psi_{uq}, \psi_{ur})$ is obtained by equating similar terms in the equation $Du = \Psi(x, u)w_u$. One easily verifies that $\psi_{uq} = u_3$, and

$$\psi_{up} = \psi_{uq} = (u_1, u_1\Delta\alpha, u_2)$$

The adaptive control law is then computed using Eq. (21). Computing the derivative of ω_d according to Eq. (17), one finds that $\psi_{2d} = 0$. Thus using Eq. (21), $\psi_{2a} = \psi_1$, and Eq. (25), the parameter update laws are given by

$$(\dot{\hat{w}}_p, \dot{\hat{w}}_q, \dot{\hat{w}}_r) = L_2^{-1}\left(\psi_p^T\tilde{\omega}_1, \psi_q^T\tilde{\omega}_2, \psi_r^T\tilde{\omega}_3\right)$$
$$(\dot{\hat{w}}_{u1}, \dot{\hat{w}}_{u2}, \dot{\hat{w}}_{u3}) = L_3^{-1}\left(\psi_{up}^T\tilde{\omega}_1, \psi_{uq}^T\tilde{\omega}_2, \psi_{ur}^T\tilde{\omega}_3\right) \tag{38}$$

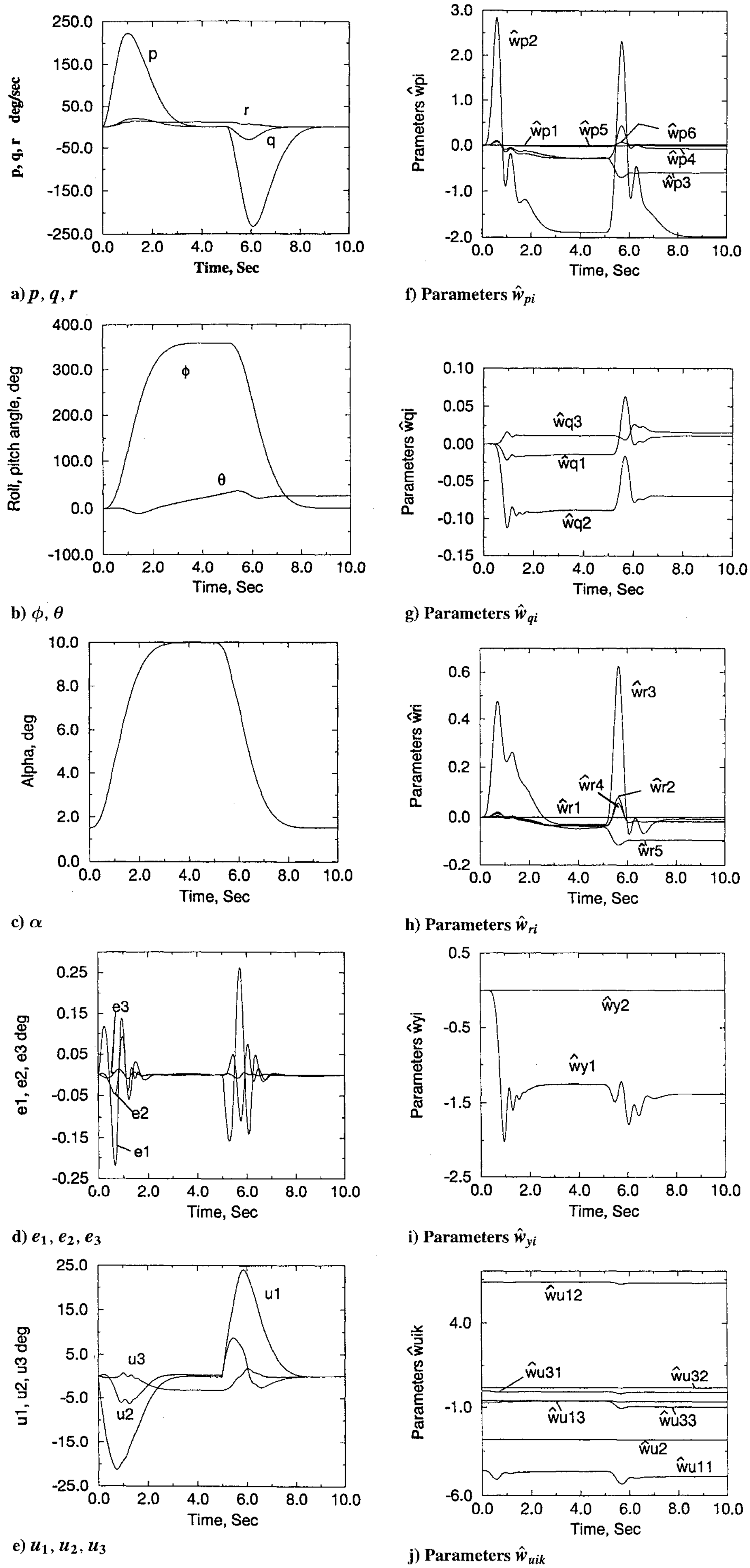
where $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \tilde{\omega}_3)^T$. The update law for $\hat{w}_1$ is also computed using Eq. (21). The command generator of order 3 is sufficient in this case. This completes the controller design.

Remark 3: The adaptive controller for controlling $y = (\phi, \theta, \beta)^T$ is similarly designed following the steps of this section. In this case $y = (\phi, \theta, \beta)^T$, $y_c = (\phi_c, \theta_c, \beta_c)^T$, $e = (y - y_c)$, and $y^* = (\phi^*, \theta^*, \beta^*)^T$, the desired terminal vector. The structure of the controller is similar. (The details of derivation are not given here.)

VI. Simulation Results

In this section, simulation results are presented for the aircraft model studied in Refs. 23 and 24 for the two flight conditions, namely, for flight condition 1 (FC 1) ($M = 0.9$, $H = 20{,}000$ ft) and for FC 2 ($M = 0.7$, $H = 0$). The complete set of aerodynamic parameters is provided in Ref. 24, and $\alpha_0 = 1.5$ deg and $\theta_0 = 0$. Nonzero parameters $y_{\delta a}$, and $z_{\delta e}$ are retained in the aircraft model to include the effect of control forces. Since the value of $y_{\delta r}$ is not given in Ref. 24, it is taken to be zero.

The initial conditions chosen for the command generator are $y_c(0) = \dot{y}_c(0) = \ddot{y}_c(0) = 0$, except $\alpha_c(0) = 1.5$ deg. The selected gains and matrices are $c_{11} = 12.726$, $c_{22} = 5$, $K_0 = 81I$, and $L_i = 10I$, and the poles of the command generator were set at -1.5, $-1.5 + j1.06$, $-1.5 - j1.06$ in the complex plane, where I denotes an identity matrix of appropriate dimension. These control parameters were selected after observing the simulated responses.



a) p, q, r

b) ϕ, θ

c) α

d) e_1, e_2, e_3

e) u_1, u_2, u_3

f) Parameters $\hat{w}_{pi}$

g) Parameters $\hat{w}_{qi}$

h) Parameters $\hat{w}_{ri}$

i) Parameters $\hat{w}_{yi}$

j) Parameters $\hat{w}_{uik}$

Fig. 1 Control of (ϕ, α, β): FC 1.

The initial conditions for the estimated parameter vector $\hat{w}(0) = [\hat{w}_1(0)^T, \hat{w}_2(0)^T]^T$ was arbitrarily set to zero, and the initial value of $\hat{w}_u(0)$ was assumed to be only 10% of value given at FC 1. These are large perturbations in the parameters for controller design. Let the equilibrium state of the aircraft be $x^* = (0, 0, 0, 1.5\text{ deg}, 0, 0, 0)^T$. In this section, the peak values of $\boldsymbol{u}$ and $\boldsymbol{e}$ are denoted as $\boldsymbol{u}_m$ and $\boldsymbol{e}_m$, respectively.

A. Control of (ϕ, α, β)

For simulation a reference trajectory was generated using a piecewise constant value of y^* as $y^*(t) = (\phi^*, \alpha^*, \beta^*)^T = (360, 10, 0)^T$ (deg) for $t \in [0, 5)$, and $y^*(t) = (0, 1.5, 0)^T$ (deg) for $t \geq 5$. Thus it is desired to roll the aircraft 360 (deg) and then roll back to $\phi = 0$, and the angle of attack is to be controlled to 10 (deg) and then to the equilibrium value, but the sideslip angle is to be kept close to zero degree. Simulation results were obtained using the parameters of aircraft for FC 1. Selected responses are shown in Figs. 1a–1j. A maximum roll rate of $p = 235.5$ (deg/s) was obtained giving a large roll-coupling effect (Fig. 1a). In spite of large parameter errors in the aerodynamic parameters, smooth roll angle and angle-of-attack control was accomplished (Figs. 1b and 1c). Only a small tracking error in the transient period was observed, and this converged to zero (Fig. 1d). The maximum error was $\boldsymbol{e}_m = (0.26, 0.11, 0.013)^T$ (deg). The adaptive controller is found to be robust with respect to the control forces, which were neglected in design. The control magnitudes required for maneuver were reasonable, Fig. 1e. The maximum magnitudes of control surface deflections were found to be $\boldsymbol{u}_m = (23.9, 8.7, 3.6)^T$ (deg). The plots of the estimated values of the aerodynamic parameters $\hat{w}_p$, $\hat{w}_q$, and $\hat{w}_r$ (Figs. 1f–1h); $\hat{w}_y$ (Fig. 1i); and $\hat{w}_u$ (Fig. 1j) show that these parameters finally converge to some constant values that differ from their actual values of FC 1. The reason for this is that the estimated parameters converge to their actual values only when the signals in the adaptation law are persistently exciting.[22] Moreover, these estimated parameters converge to points that depend on their chosen initial values and the command trajectory.

Simulation was also performed using the aircraft model for FC 2 and the adaptive controller. The feedback gains and the initial values of the parameters $\hat{w}(0)$ and $\hat{w}_u(0)$ were not modified. Smooth responses were observed in this case also. Figure 2 shows the tracking error. The maximum error was $\boldsymbol{e}_m = (0.24, 0.117, 0.013)^T$ (deg).

B. Control of (ϕ, θ, β)

A controller was designed as indicated in Remark 3, and simulation was done to control the output $y = (\phi, \theta, \beta)^T$. It was desired to control the roll and pitch angles from the initial value (0,0) to (60, 40) (deg), and then to (0, 0). Furthermore, the sideslip angle was to be regulated close to zero. For generating the command trajectory $y_c = (\phi_c, \theta_c, \beta)^T$, we set y^* to $y^*(t) = (60, 40, 0)$ deg for $t \in [0, 5)$ (s), and $y^* = 0$ for $t \geq 5$ (s) in Eq. (5). Selected responses of the aircraft model for FC 2 are shown in Figs. 3a–3c. Smooth responses for pitch and roll angles (Fig. 3a) were obtained. The tracking error (Fig. 3b) converged to zero. The maximum error was $\boldsymbol{e}_m = (0.026, 0.016, 0.0074)$, and the control magnitude was less than 6 deg. The estimated parameters converged to constant values. (Only the plot of $\hat{w}_p$ is shown in Fig. 3c.)

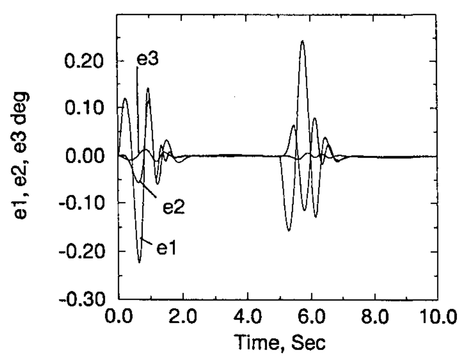
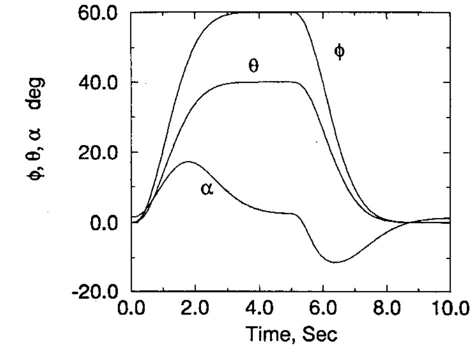
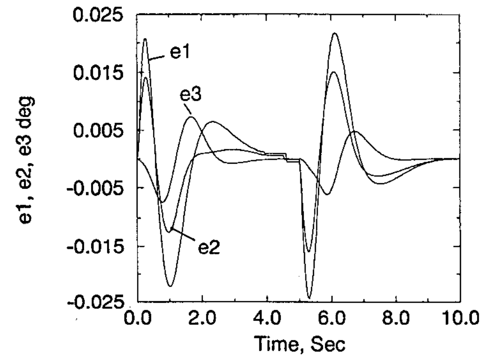


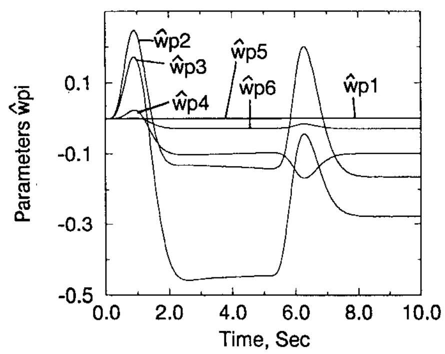
Fig. 2 Control of (ϕ, α, β): FC 2, e_1, e_2, e_3.



a) ϕ, θ, α



b) e_1, e_2, e_3



c) Parameters $\hat{w}_{pi}$

Fig. 3 Control of (ϕ, θ, β): FC 2.

Simulation was also performed for the model of aircraft for FC 1 using the same controller. Again, smooth responses somewhat similar to those in Fig. 3 were obtained. (These results are not shown here.)

VII. Conclusion

In this paper adaptive control of a class of nonlinear input–output feedback linearizable systems using state variable feedback was considered. An adaptive control law was derived such that the trajectory asymptotically converges to a hypersurface in the state space. The parameters of the hypersurface were chosen such that for any trajectory confined to this surface, the tracking error converges to zero. The tracking error is the output of a stable filter. The parameters of this filter can be chosen to shape the tracking error responses. These results were applied to the design of control systems for the trajectory control of the sets of output variables (roll angle, angle of attack, sideslip angle) and roll angle, pitch angle, and sideslip angle using aileron, rudder, and elevator control surfaces. Extensive simulation was performed that showed that nonlinear precise roll-coupled maneuvers of aircraft can be accomplished in the closed-loop system in spite of large uncertainty in the aerodynamic parameters.

References

[1] Isidori, A., *Nonlinear Control Systems*, Springer–Verlag, Berlin, 1985, pp. 175–185.

[2]Nijmeijer, H., and Van Der Schaft, A. J., *Nonlinear Dynamical Control Systems*, Springer–Verlag, New York, 1990.

[3]Sastry, S. S., and Isidori, A.,"Adaptive Control of Linearizable Systems," *IEEE Transactions on Automatic Control*, Vol. 34, No. 11, 1989, pp. 1123–1131.

[4]Ziang, Z. P., and Praly, L., "Iterative Designs of Adaptive Controllers for Systems with Nonlinear Integrators," *Proceedings of the 30th IEEE Conference on Decision and Control* (Brighton, England, UK), Inst. of Electrical and Electronics Engineers, 1991, pp. 2482–2487.

[5]Kanellakopoulos, I., Kokotovic, P. V., and Morse, A. S.,"Systematic Design of Adaptive Controllers for Feedback Linearizable Systems,"*IEEE Transactions on Automatic Control*, Vol. 36, No. 11, 1991, pp. 1241–1253.

[6]Kristic, M., Kanellakopoulos, I., and Kokotovic, P. V.,"Adaptive Nonlinear Control Without Overparametrization," *Systems and Control Letters*, Vol. 19, 1992, pp. 177–185.

[7]Ghanadan, R., and Blankenship, G. L., "Adaptive Approximate Tracking and Regulation of Nonlinear Systems," *Proceedings of the IEEE Conference on Decision and Control* (San Antonio, TX), Inst. of Electrical and Electronics Engineers, Piscataway, NJ, 1993, pp. 2654–2659.

[8]Kwan, C. M., and Lewis, F. L., "Robust Backstepping Control of Nonlinear Systems Using Neural Networks," *Proceedings of European Control Conference* (Rome), 1995, pp. 2772–2777.

[9]Meyer, G., Su, R., and Hunt, L. R., "Application of Nonlinear Transformations to Automatic Flight Control," *Automatica*, Vol. 20, No. 1, 1984, pp. 103–107.

[10]Menon, P. K. A., Badgett, M. E., Walker, R. A., and Duke, E. L., "Nonlinear Flight Test Trajectory Controllers for Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 1, 1987, pp. 67–72.

[11]Lane, S. H., and Stengel, R. F., "Flight Control Design Using Nonlinear Inverse Dynamics," *Automatica*, Vol. 24. No. 4, 1988, pp. 471–484.

[12]Snell, S. A., Enns, D. F., and Garrard, W. L., "Nonlinear Inversion Flight Control for a Supermaneuverable Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 4, 1992, pp. 976–984.

[13]Azam, M., and Singh, S. N., "Invertibility and Trajectory Control for Nonlinear Maneuvers of Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 1, 1994, pp. 192–200.

[14]Elgersma, M. R., "Control of Nonlinear Systems and Application to Aircraft," Ph.D. Dissertation, Dept. of Electrical Engineering, Univ. of Minnesota, Minneapolis, MN, April 1988.

[15]Hauser, J. E., Sastry, S., and Meyer, G., "Nonlinear Control Design for Slightly Nonminimum Phase System: Application to V/STOL Aircraft," *Automatica*, Vol. 28, No. 4, 1922, pp. 665–680.

[16]Khan, M. A., and Lu, P., "New Technique for Control of Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 5, 1994, pp. 1055–1060.

[17]Enns, D.,"Robustness of Dynamic Inversion vs μ-Synthesis:Lateral Directional Flight Control Example," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Washington, DC, 1990, pp. 210–222 (AIAA Paper 90-3338).

[18]Reiner, J., Balas, G. J., and Garrard, W. L., "Robust Dynamic Inversion for Control of Highly Maneuverable Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 1, 1995, pp. 18–24.

[19]Adams, R. J., and Banda, S. S., "Robust Flight Control Design Using Dynamic Inversion and Structured Singular Value Synthesis," *IEEE Transactions on Control Systems Technology*, Vol. 1, No. 2, 1993, pp. 80–92.

[20]Slotine, J.-J. E., and Li, W., *Applied Nonlinear Control*, Prentice–Hall, Englewood Cliffs, NJ, 1991, pp. 123, 124.

[21]Singh, S. N., "Asymptotically Decoupled Discontinuous Control of Systems and Nonlinear Aircraft Maneuver," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 25, No. 3, 1989, pp. 380–391.

[22]Narendra, K. S., and Annaswamy, A., *Stable Adaptive Systems*, Prentice–Hall, Englewood Cliffs, NJ, 1989, pp. 246–248, 308–310.

[23]Hacker, T., and Oprisiu, C., "A Discussion of the Roll-Coupled Problem," *Progress in Aerospace Science*, Vol. 15, Pergamon, Oxford, England, UK, 1974, pp. 151–180.

[24]Rhoads, D. W., and Schuler, T. M., "A Theoretical and Experimental Study of Airplane Dynamics in Large-Disturbance Maneuvers," *Journal of the Astronautical Sciences*, Vol. 24, July 1957, pp. 507–532.